The influence of washing machine-leg hardness on its dynamics response within component-mode synthesis techniques

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Abstract

In this paper, we investigate a washing machine's dynamic response. Using an experimental modal analysis, the dependence of the first two natural frequencies on the hardness of leg rubber is demonstrated. In order to model this behaviour, a complete washing-machine numerical model is developed, including a detailed model of the leg. A simple linear leg model is proposed, which accounts for the contact conditions and enables an implicit analysis. The model is validated, based on two measurements with different leg configurations. Additionally, the component-mode synthesis methods are proposed. They allow separate treatment of the washing machine's legs and cabinet, as well as reducing the model order. The four model-reduction techniques are compared with the classic finite-element method. It is shown that the component-mode synthesis methods enable fast recalculation times for the modified substructures, while the remaining structure is calculated only once. This leads to a computationally efficient analysis. A comparison of the results shows good agreement between the component-mode synthesis methods and the classic finiteelement method.

1 Introduction

The design and research of home-appliance products focuses not only on efficiency and performance, but also on quiet and user-friendly products. One of the best-known sources of household noise and vibration is the washing machine. The scientific community and industry have carried out several studies related to modelling and reducing noise and vibration. Türkay *et al.* [25] presented formulations and implementations for the optimisation of a suspension design. In [17] Jakšič *et al.* studied the theoretical aspects of a planar, non-linear, centrifugally excited, oscillatory system in its steady-state domain. The theoretical approach was verified experimentally on complex washing-machine dynamics by Boltežar *et al.* [6]. Conrad and Soedel [9] studied the influence of weight reduction on the walking stability of a washing machine, Chen *et al.* [7] researched stability of a vertical axis washing machine with a hydraulic balancer, Chen *et al.* [8] studied the steady-state response using a new approach and a method for getting a smaller deflection angle and Bae *et al.* [3] made dynamic analysis of an automatic washing machine with a hydraulic balancer. An analysis of the sound quality due to impacts is presented in [18], while velocity control is studied in [5].

The washing machine is a complex system; therefore, advanced numerical methods must be applied to model the system dynamics. Classically, the analysis is performed with the use of the finite-element method (FEM), where the model is represented by a dense mesh with a large number of degrees of freedom (DOF). Here, an alternative to the classic finite-element analysis is presented by applying substructuring techniques, which consist of a model reduction and a substructure assembly, also known as Component-Mode Synthesis. The modelreduction techniques were first introduced in 1965, when the Guayan method was presented [16]. Soon after the Craig Bampton [10] (1968), MacNeal [19] (1971), Rubin^[24] (1975) and Craig-Chang methods ^[11] (1977) followed. Note that the Craig-Chang and Rubin methods have the same reduced equations of motion, but with a different derivation procedure. A more recent method is the Dual Craig-Bampton method [23] (2004). These methods make it possible to reduce the model order and the assembly of substructures. The substructures can be analysed independently of the remaining structure. Moreover, the techniques also allow a quick re-analysis of the modifications. The model reduction reduces the system matrices and therefore allows faster computations, especially when a transient response (i.e., time integration) is required. These advantages are particularly useful when large models are analysed and only certain subcomponents are changing in the model.

In this paper a structural model of a washing machine with a detailed model of the legs is presented. A new, numerical model of the leg structure is proposed that accounts for the deformations as well as the contact conditions. The model is based on the rubber-metal contact formulation presented by Medina et. al. [20], which presents the influence of the material properties and the contact area on the tangential stiffness. For practical reasons a measurement of the contact area is not always possible, and it is demonstrated in this paper that the tangential stiffness can also be obtained through a single measurement of the modal parameters. The presented approach introduces the so-called shear modulus correction factor k_{corr} that is calculated based on minimizing the difference between the corresponding measured and numerically obtained natural frequencies of the whole washing-machine structure. It is demonstrated that the proposed identification process for the shear modulus correction factor is reliable and accurate and may be applied to leg structures with different rubber hardnesses. Due to the nature of the problem, where a small leg model is of interest and the rest of the structure remains unchanged, substructuring techniques are proposed to analyse these modifications. In this paper four substructuring techniques are used and compared with the results obtained using the classic finite-element method. Finally, the numerical model is validated experimentally.

The article is organised as follows. The second section presents the numerical model of the leg and the obtained results with the classic finite-element method. The third section presents a determination of the leg parameters and the experimental validation of the numerical model. In the fourth section the four substructuring techniques are presented and the comparison with the FEM method is made. In the last section a summary and the contributions are presented.

2 Numerical model

The numerical model of the washing machine is presented in Figure 1. It consists of a mesh of 3D solid (SOLID45) and 2D shell (SHELL181) elements and is constructed in a classic FEM manner. The cover and legs consist of solid elements, while the cabinet is modelled using shell elements.



Figure 1: Numerical model

The legs are treated in more detail, because of their impact on the system dynamics, most importantly the position of the first and the second natural frequency. The legs consist of a bolt with a nut, a rubber foot and a steel end (Figure 2). The legs differ in terms of the shape of the feet and the material, which is usually rubber. In operation they are mainly exposed to shear strain where the contact between the rubber feet and the ground has to be accounted for. The development of a detailed leg model represents a complex task as the friction conditions (static friction), the large deformations and the damping have to be accounted for. Thus, a simpler linear leg model is proposed, which includes the deformations as well as the friction conditions. It enables an implicit analysis and in this way the computation of the modal parameters. The model is defined as a bolt with a nut (solid elements), a steel end (solid elements), a rubber foot (solid elements) and the ground (Figure 3). The coupling conditions between the leg and the cabinet as well as the ground are modelled using fully constrained DoFs.



Figure 2: Waschine maschine's leg.

As shown by *Medina et. al.* [16] the modelling of rough elastic contacts can be expressed in terms of the tangential stiffness:

$$k_T = \frac{4E}{(1+\nu)(2-\nu)} \sum_i a_i = \frac{8G}{(2-\nu)} \sum_i a_i,$$
(1)

where ν is the Poisson's ratio, E is the Young's modulus and a_i is the radius of the contact patch, defined by:

$$a_i = \left(\frac{3\,P_i\,R_i}{4\,E^*}\right)^{\frac{1}{3}},\tag{2}$$

where P_i represents the normal load carried by an asperity *i*, R_i is the radius of curvature of the asperity *i* and E^* is the expression for the equivalent modulus of the contacting surfaces:

$$E^* = \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]^{-1},\tag{3}$$

where ν is the Poisson's ratio and E the Young's modulus of the materials in contact and the subscripts refer to each of the contacting bodies. If one of the bodies is assumed to be rigid, Eq. (3) simplifies to:

$$E^* = \frac{E}{1 - \nu^2}.\tag{4}$$

However, we can write

$$\sum_{i} a_{i} = n \,\overline{a},\tag{5}$$

where n is the number of asperities in contact and \overline{a} is the mean radius of the asperity contact. They assume that the asperity heights are distributed normally with a standard deviation of σ and have a constant radius of curvature R. Under these conditions, they demonstrate that \overline{a} is approximately independent of the normal load, whereas n is proportional to it. According to the analytical model [20], the elastic modulus and asperity radius should influence the

contact area and the number of asperities, but have no direct influence on tangential stiffness when exposed to low pressures. When analysing different elastic materials with the same Poisson's ratio ν and an assumed equal contact area $\sum a_i$, a linear dependence of tangential stiffness and shear modulus is observed. In this paper this assumption is adopted by introducing the correction factor k_{corr} , which models the tangential stiffness without measuring the contact area. The correction factor is determined based on the value of the first two natural frequencies of the whole washing machine structure. In order to simulate the



Figure 3: Numerical model of a leg.

leg behaviour, orthotropic material properties are proposed for the rubber foot. The rubber foot is described by the rubber's elastic modulus E and the shear modulus G. The elastic modulus is evaluated experimentally, while the shear modulus is computed as:

$$G = \frac{E}{2\left(1+\nu\right)}\tag{6}$$

where ν is the Poissons's ratio. The shear moduli for all three planes are defined as:

$$G_{xy} = G/k_{corr}$$

$$G_{xz} = G$$

$$G_{yz} = G/k_{corr}$$
(7)

where k_{corr} is a shear-modulus correction factor and is determined experimentally. It simply simulates the rubber-metal (ground) contact conditions in the xy and yz directions, accounting for static friction conditions, which are in general difficult to model. Hence, the assumption of orthotropic material properties allows the modelling of complex contact conditions in the form of a simple linear model. The linear assumption is valid due to small deformations of the rubber feet during operation and due to the assumption of an equal contact area $\sum a_i$ [20]. The procedure is similar as in [22], where the friction contacts between laminas in laminated structures were modelled.

In the next section the algorithm to obtain the properties of the rubber foot material and the validation of the numerical model are presented.

3 The determination of leg's material parameters and the validation of the numerical model

The numerical model of the leg is validated in the first stage with an experiment. The experimental set-up enabled a measurement of the natural frequencies and the corresponding mode shapes with the experimental modal analysis (EMA). The washing machine was positioned on steel plate in a closed environment with a temperature of 25°C. The measurements of the washing machine's dynamic properties were made on a non-operating machine with an impact hammer and a roving 3-axial piezoelectric accelerometer at 364 points on the cabinet (Figure 4). The sampling rate was set to 25600 samples per second, while the window length was 102400 samples. The measurements were made with OpenModal software [2]. Note, that when the washing machine is in operation, the position of the natural frequencies remains constant. This enables the use of a numerical model in the operating and non-operating mode, unless the legs start to lose contact with the ground. In this case model updating of the reduced leg model is required in order to represent the changing boundary conditions.



Figure 4: Experimental setup

Two measurements were conducted with different leg configurations in terms of the hardness of the rubber feet [1]: Shore 70A and Shore 85A.

From the measured natural frequencies and their corresponding mode shapes it was observed that the first and second natural frequencies strongly depend on the leg's stiffness (Figures 5a and 5b).

The first mode shape represents the forward-backward swinging of the washing machine and the second mode shape represents the left-right swinging due to deformations of the legs. The stiffness of the rubber leg material directly influences the swinging motion and therefore alters the first two natural frequencies. At higher frequencies this influence is considerably smaller. Hence, the shear correction factor k_{corr} describes, in a simple way, the deformations and friction conditions between the ground, the rubber and the steel end. It is



Figure 5: Washing-machine mode shapes; a) First mode shape, b) Second mode shape.

determined by an optimisation process, minimizing the following expression:

$$|f_{num,1}(k_{corr}) - f_{exp,1}| < \varepsilon \tag{8}$$

where $f_{num,1}(k_{corr})$ is the first numerical natural frequency, which depends on the shear-modulus correction factor k_{corr} , and $f_{exp,1}$ is the first experimental natural frequency. The optimisation process is complete when the numerical and measured natural frequencies overlap within an allowed tolerance ε .

Here, the first configuration with the leg rubber hardness of the Shore 70A served as reference to obtain the shear-modulus correction factor k_{corr} and therefore to define the orthotropic material properties of the rubber feet. Based on a measurement the shear correction factor was computed to be $k_{corr} = 2.7$. Once the shear correction factor was obtained, the shear modulus G_{yz} for legs with the rubber hardness of the Shore 85A could be predicted. The material parameters used to compute the modal parameters are given in Table 1.

The comparison of the numerical (FEM) and the experimental natural frequencies (Table 2), for the Shore 70A leg rubber hardness shows good agreement. This confirms the appropriateness of the procedure to determine the rubber's shear modulus. The second measurement, with the Shore 85A leg rubber hardness, served for a validation of the leg's numerical model, since the correction factor was already determined from the first measurement (Shore 70A). Based on the correction factor k_{corr} and the rubber's elastic modulus E, the obtained natural frequencies show a good agreement with the results obtained using the

Table 1: Leg material properties.

	Shore 70A	Shore 85A
E [MPa]	2.00	2.31
u	0.33	0.33
$G = G_{xz}$ [MPa]	0.73	0.88
$G_{yz} = G_{xy} = G/2.7$ [MPa]	0.27	0.33

EMA. Note that some natural frequencies could not be detected experimentally, since they could not be excited.

		Shore 70A	Ι	Shore 85A			
#	FEM [Hz]	Exp. [Hz]	Rel. Err. [%]	FEM [Hz]	Exp. [Hz]	Rel. Err. $[\%]$	
1	29.538	29.6	0.209	31.303	31.5	0.625	
2	35.274	35.3	0.0737	36.846	36.7	0.398	
3	44.691	44.2	1.111	44.723	44.2	1.183	
4	53.188	53.6	0.769	53.280	54.5	2.239	
5	56.633	56.5	0.235	59.281	56.5	4.922	
6	60.213	/	/	60.299	/	/	
7	61.216	61.4	0.300	61.997	61.4	0.972	
8	66.771	65.7	1.630	67.461	66.5	1.445	
	/	71.1	/	/	70.2	/	
9	78.446	78.7	0.323	78.540	78.8	0.330	
10	81.373	/	/	82.973	/	/	
11	82.877	/	/	83.151	/	/	
12	83.891	/	/	84.177	/	/	
13	84.706	85.1	0.463	84.927	85.8	1.017	
14	88.145	/	/	88.554	/	/	
15	89.782	/	/	89.782	/	/	
16	95.657	93.5	2.307	97.874	95.0	3.025	

Table 2: Numerical and experimental washing-machine natural frequencies in the 0-100 Hz range with a relative error for two different leg-rubber hardnesses.

When changing the first natural frequency by 2 Hz or 120 rpm, the operating range also changes. In our case the first natural frequency is around 29.6 Hz or 1776 rpm, which is near the maximum operating speed of 1600. Hence, if the first natural frequency is increased by 120 rpm, the system dynamic response decreases, which results in lower noise and vibration.

The analysis of the whole washing machine is a complex task. However, since only the leg model is changing, the substructuring techniques offer an alternative way to predict the dynamic response of such systems.

4 Substructuring techniques

The classic finite-element approach requires a large number of nodes (elements), which leads to large models and long computation times. In order to reduce the time a coarser mesh needs to be applied, which is not always possible due to the convergence of the solution. A possible alternative is the model-reduction techniques applied to substructures with a final assembly. This is known as component-mode synthesis. The two main advantages are the reduced number of degrees of freedom (DOF) and the possibility to recompute only the changing substructure (i.e. legs). Another advantage is the use of reduced models in the explicit analysis, for instance the simulation of the washing machine's start-up.

For these reasons the washing machine and the four legs are treated as substructures. Overall, the model is defined by 231 402 DOF, where the four legs have 3144 DOF and the rest of the structure consists of 228258 DOF.

4.1 Model reduction

Model reduction retains the dense finite-element mesh, but replaces the physical degrees of freedom by a much smaller set of generalized degrees of freedom. This is done by modal superposition and truncation.

In our case four different model-reduction methods are used: the Craig-Bampton [10], the Rubin [24], the MacNeal [19] and the Dual Craig-Bampton [23]. A good overall step-by-step description of the methods can be found in [26]. The methods consist of a reduction basis containing static and a limited number of vibration modes. The static modes can be further divided into the constraint, attachment and residual attachment modes. The vibration modes are divided into the free-interface, rigid-body and fixed-interface modes. A detailed description of the above-mentioned modes is found in [26], [15] and [13].

The model-reduction techniques are closely connected to the substructuring field, where a substructure dynamical model is defined as:

$$\mathbf{M}^{(s)}\ddot{\mathbf{u}}^{(s)}(t) + \mathbf{C}^{(s)}\dot{\mathbf{u}}^{(s)}(t) + \mathbf{K}^{(s)}\mathbf{u}^{(s)}(t) = \mathbf{f}^{(s)}(t) + \mathbf{g}^{(s)}(t),$$
(9)

The matrices $\mathbf{M}^{(s)}$, $\mathbf{C}^{(s)}$ and $\mathbf{K}^{(s)}$ represent the mass, damping and stiffness matrix of a substructure s, $\mathbf{u}^{(s)}(t)$ is the displacement vector, $\mathbf{f}^{(s)}$ is the external excitation vector and $\mathbf{g}^{(s)}$ is the vector of connection forces with the surrounding substructures.

4.1.1 Craig-Bampton method

The Craig-Bampton method [10] divides the physical DOF \mathbf{u} into the internal \mathbf{u}_i and the boundary DOF \mathbf{u}_b , which gives Eq. (9) the following shape:

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix} \left\{ \begin{array}{c} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \end{array} \right\} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{u}_i \\ \mathbf{u}_b \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{f}_i \\ \mathbf{f}_b \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{g}_i \\ \mathbf{g}_b \end{array} \right\}, \quad (10)$$

where the index *i* denotes the internal DOF and *b* the boundary DOF. Note that the internal excitation forces \mathbf{g}_i are assumed to be $\mathbf{0}$, since there is no contact with the neighbouring substructures.

The internal DOF are approximated as:

$$\mathbf{u}_i \approx \Psi_c \, \mathbf{u}_b + \Phi_i \, \boldsymbol{\eta}_i \tag{11}$$

Here, Ψ_c are the static constraint modes and Φ_i are a reduced set of fixed interface vibration modes with the corresponding modal DOF η_i . Hence, the reduction basis is the following:

$$\left\{ \begin{array}{c} \mathbf{u}_i \\ \mathbf{u}_b \end{array} \right\} \approx \left[\begin{array}{c} \boldsymbol{\Phi}_i & \boldsymbol{\Psi}_c \\ \mathbf{0} & \mathbf{I} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{\eta}_i \\ \mathbf{u}_b \end{array} \right\} = \mathbf{R}_{CB} \, \mathbf{q}_{CB}$$
 (12)

If Eq. (12) is inserted into Eq. (10) and the orthogonality between the vibration modes with respect to the mass or stiffness matrix [15] is taken into account, the following reduced equations of motion are obtained:

$$\begin{bmatrix} \mathbf{I} & \mathbf{M}_{\phi b} \\ \mathbf{M}_{b\phi} & \tilde{\mathbf{M}}_{bb} \end{bmatrix} \left\{ \begin{array}{c} \ddot{\boldsymbol{\eta}}_i \\ \ddot{\mathbf{u}}_b \end{array} \right\} + \left[\begin{array}{c} \boldsymbol{\Omega}_i^2 & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{K}}_{bb} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{\eta}_i \\ \mathbf{u}_b \end{array} \right\} = \left\{ \begin{array}{c} \tilde{\mathbf{f}}_i \\ \tilde{\mathbf{f}}_b \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{g}_b \end{array} \right\}, \quad (13)$$

where:

$$\begin{aligned}
\tilde{\mathbf{K}}_{bb} &= \mathbf{K}_{bb} - \mathbf{K}_{bi} \, \mathbf{K}_{ii}^{-1} \, \mathbf{K}_{ib} \\
\tilde{\mathbf{M}}_{bb} &= \mathbf{M}_{bb} - \mathbf{M}_{bi} \, \mathbf{K}_{ii}^{-1} \, \mathbf{K}_{ib} - \mathbf{K}_{bi} \, \mathbf{K}_{ii}^{-1} \, \mathbf{M}_{ib} + \mathbf{K}_{bi} \, \mathbf{K}_{ii}^{-1} \, \mathbf{M}_{ii} \, \mathbf{K}_{ii}^{-1} \, \mathbf{K}_{ib} \\
&= \mathbf{M}_{bb} - \mathbf{M}_{bi} \, \Psi_c - \Psi_c^{\mathrm{T}} \, \mathbf{M}_{ib} + \Psi_c^{\mathrm{T}} \, \mathbf{M}_{ii} \, \Psi_c \\
\mathbf{M}_{\phi b} &= \Phi_i^{\mathrm{T}} (\mathbf{M}_{ib} - \mathbf{M}_{ii} \, \mathbf{K}_{ii}^{-1} \, \mathbf{K}_{ib}) \\
\mathbf{M}_{b\phi} &= \mathbf{M}_{\phi b}^{\mathrm{T}} \\
\tilde{\mathbf{f}}_i &= \Phi_i^{\mathrm{T}} \, \mathbf{f}_i \\
\tilde{\mathbf{f}}_b &= \mathbf{f}_b - \mathbf{K}_{bi} \, \mathbf{K}_{ii}^{-1} \, \mathbf{f}_i = \Psi_c^{\mathrm{T}} \, \mathbf{f}_i
\end{aligned} \tag{14}$$

Here, Ω_i^2 represents a diagonal matrix of squared fixed-interface frequencies $\omega_{i,j}^2$.

4.1.2 Rubin and MacNeal methods

Both the Rubin [24] and MacNeal methods [19] have the same reduction basis containing free-interface modes Φ_f with the corresponding modal DOF η_f , rigid-body modes Φ_r with the corresponding modal DOF η_r and residual attachment modes Ψ_r . They approximate the displacement vector as:

$$\mathbf{u} \approx \boldsymbol{\Psi}_r \, \mathbf{g}_b + \boldsymbol{\Phi}_r \, \boldsymbol{\eta}_r + \boldsymbol{\Phi}_f \, \boldsymbol{\eta}_f \tag{15}$$

Inserting Eq. (15) into Eq. (9) and accounting for the orthogonality between the vibration modes, the rigid-body modes and their combination [15], gives the following form:

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{r,bb} \end{bmatrix} \left\{ \begin{array}{c} \ddot{\boldsymbol{\eta}}_r \\ \ddot{\boldsymbol{\eta}}_f \\ \ddot{\mathbf{g}}_b \end{array} \right\} + \left[\begin{array}{c} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_f^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{r,bb} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{\eta}_r \\ \boldsymbol{\eta}_f \\ \mathbf{g}_b \end{array} \right\} = \left\{ \begin{array}{c} \boldsymbol{\Phi}_r^{\mathrm{T}} \\ \boldsymbol{\Phi}_f^{\mathrm{T}} \\ \boldsymbol{\Psi}_r^{\mathrm{T}} \end{array} \right\} \mathbf{f} + \left\{ \begin{array}{c} \boldsymbol{\Phi}_r^{\mathrm{T}} \\ \boldsymbol{\Phi}_f^{\mathrm{T}} \\ \boldsymbol{\Psi}_r^{\mathrm{T}} \end{array} \right\} \mathbf{g}$$
(16)

where:

$$\mathbf{G}_{r,bb} = \mathbf{\Psi}_{r}^{\mathrm{T}} \mathbf{K} \mathbf{\Psi}_{r} = \mathbf{A} \mathbf{G}_{r} \mathbf{A}^{\mathrm{T}}
\mathbf{K}_{r,bb} = \mathbf{G}_{r,bb}^{-1}
\mathbf{M}_{r,bb} = \mathbf{\Psi}_{r}^{\mathrm{T}} \mathbf{M} \mathbf{\Psi}_{r}$$
(17)

A is a boolean matrix selecting the interface DOF and \mathbf{G}_r is the residual flexibility matrix, which is obtained from the residual attachment modes [26, 13]. Both methods apply a second transformation in order to transform the interface force DOF \mathbf{g}_b to the interface displacements \mathbf{u}_b . This is done by pre-multiplying Eq. (15) by the boolean matrix \mathbf{A} :

$$\mathbf{u}_{b} = \mathbf{A} \,\mathbf{u} = \mathbf{A} (\Psi_{r} \,\mathbf{g}_{b} + \Phi_{r} \,\boldsymbol{\eta}_{r} + \Phi_{f} \,\boldsymbol{\eta}_{f}) = \mathbf{G}_{r,bb} \,\mathbf{g}_{b} + \Phi_{r|b} \,\boldsymbol{\eta}_{r} + \Phi_{f|b} \,\boldsymbol{\eta}_{f} \quad (18)$$

which defines the second transformation as:

$$\left\{ \begin{array}{c} \boldsymbol{\eta}_r \\ \boldsymbol{\eta}_f \\ \mathbf{g}_b \end{array} \right\} = \left[\begin{array}{ccc} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{r|b} & -\mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{f|b} & \mathbf{K}_{r,bb} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{\eta}_r \\ \boldsymbol{\eta}_f \\ \mathbf{u}_b \end{array} \right\}$$
(19)

The Rubin method is defined by inserting Eq. (19) into Eq. (16) and the Rubin reduced equations of motion are obtained:

$$\begin{bmatrix} \mathbf{I} + \mathbf{\Phi}_{r|b}^{\mathrm{T}} \overline{\mathbf{M}}_{r} \mathbf{\Phi}_{r|b} & \mathbf{\Phi}_{r|b}^{\mathrm{T}} \overline{\mathbf{M}}_{r} \mathbf{\Phi}_{f|b} & -\mathbf{\Phi}_{r|b}^{\mathrm{T}} \overline{\mathbf{M}}_{r} \\ \mathbf{\Phi}_{f|b}^{\mathrm{T}} \overline{\mathbf{M}}_{r} \mathbf{\Phi}_{r|b} & \mathbf{I} + \mathbf{\Phi}_{f|b}^{\mathrm{T}} \overline{\mathbf{M}}_{r} \mathbf{\Phi}_{f|b} & -\mathbf{\Phi}_{f|b}^{\mathrm{T}} \overline{\mathbf{M}}_{r} \end{bmatrix} \left\{ \begin{array}{c} \ddot{\boldsymbol{\eta}}_{r} \\ \ddot{\boldsymbol{\eta}}_{f} \\ \ddot{\boldsymbol{\eta}}_{f} \\ \ddot{\boldsymbol{\eta}}_{f} \\ \ddot{\boldsymbol{u}}_{b} \end{array} \right\} + \\ \begin{bmatrix} \mathbf{\Phi}_{r|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \mathbf{\Phi}_{r|b} & \mathbf{\Phi}_{r|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \mathbf{\Phi}_{f|b} & -\mathbf{\Phi}_{r|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \\ \mathbf{\Phi}_{f|b}^{\mathrm{T}} \mathbf{K}_{r} \mathbf{\Phi}_{r|b} & \mathbf{\Omega}_{f}^{2} + \mathbf{\Phi}_{f|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \mathbf{\Phi}_{f|b} & -\mathbf{\Phi}_{f|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \\ -\mathbf{K}_{r,bb} \mathbf{\Phi}_{r|b} & \mathbf{\Omega}_{f}^{2} + \mathbf{\Phi}_{f|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \mathbf{\Phi}_{f|b} & -\mathbf{\Phi}_{f|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \\ -\mathbf{K}_{r,bb} \mathbf{\Phi}_{r|b} & -\mathbf{K}_{r,bb} \mathbf{\Phi}_{f|b} & \mathbf{K}_{r,bb} \end{bmatrix} \left\{ \begin{array}{c} \boldsymbol{\eta}_{r} \\ \boldsymbol{\eta}_{f} \\ \mathbf{u}_{b} \end{array} \right\} = (20) \\ \left\{ \begin{array}{c} \tilde{\mathbf{f}}_{r} \\ \tilde{\mathbf{f}}_{f} \\ \tilde{\mathbf{f}}_{b} \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{0} \\ \mathbf{g}_{b} \end{array} \right\}$$

where:

$$\mathbf{M}_{r} = \mathbf{K}_{r,bb} \mathbf{M}_{r,bb}
\tilde{\mathbf{f}}_{r} = (\mathbf{\Phi}_{r}^{\mathrm{T}} - \mathbf{\Phi}_{r|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \mathbf{\Psi}_{r}^{\mathrm{T}}) \mathbf{f}
\tilde{\mathbf{f}}_{f} = (\mathbf{\Phi}_{f}^{\mathrm{T}} - \mathbf{\Phi}_{f|b}^{\mathrm{T}} \mathbf{K}_{r,bb} \mathbf{\Psi}_{r}^{\mathrm{T}}) \mathbf{f}
\tilde{\mathbf{f}}_{b} = \mathbf{K}_{r,bb} \mathbf{\Psi}_{r}^{\mathrm{T}} \mathbf{f}$$
(21)

The Rubin reduction basis can therefore be defined as follows:

$$\left\{ \begin{array}{c} \mathbf{u}_{i} \\ \mathbf{u}_{b} \end{array} \right\} \approx \left[\begin{array}{c} \boldsymbol{\Phi}_{r|i} - \boldsymbol{\Psi}_{r|i} \, \mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{r|b} & \boldsymbol{\Phi}_{f|i} - \boldsymbol{\Psi}_{r|i} \, \mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{f|b} & \boldsymbol{\Psi}_{r|i} \, \mathbf{K}_{r,bb} \\ \mathbf{0} & \mathbf{I} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{\eta}_{r} \\ \boldsymbol{\eta}_{f} \\ \mathbf{u}_{b} \end{array} \right\} = \mathbf{R}_{R} \, \mathbf{q}_{R}$$

$$(22)$$

The MacNeal method differs from the Rubin method in neglecting the residual mass term $\mathbf{M}_{r,bb}$ in Eq. (16). The procedure afterwards is similar, leading to MacNeal's reduced equations of motion:

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{cases} \ddot{\boldsymbol{\eta}}_{r} \\ \ddot{\boldsymbol{\eta}}_{f} \\ \ddot{\boldsymbol{u}}_{b} \end{cases} + \\ \begin{bmatrix} \boldsymbol{\Phi}_{r|b}^{\mathrm{T}} \, \mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{r|b} & \boldsymbol{\Phi}_{r|b}^{\mathrm{T}} \, \mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{f|b} & -\boldsymbol{\Phi}_{r|b}^{\mathrm{T}} \, \mathbf{K}_{r,bb} \\ \boldsymbol{\Phi}_{f|b}^{\mathrm{T}} \, \mathbf{K}_{r} \, \boldsymbol{\Phi}_{r|b} & \boldsymbol{\Omega}_{f}^{2} + \boldsymbol{\Phi}_{f|b}^{\mathrm{T}} \, \mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{f|b} & -\boldsymbol{\Phi}_{f|b}^{\mathrm{T}} \, \mathbf{K}_{r,bb} \\ -\mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{r|b} & -\mathbf{K}_{r,bb} \, \boldsymbol{\Phi}_{f|b} & \mathbf{K}_{r,bb} \end{bmatrix} \begin{cases} \boldsymbol{\eta}_{r} \\ \boldsymbol{\eta}_{f} \\ \mathbf{u}_{b} \end{cases} = (23) \\ \begin{cases} \tilde{\mathbf{f}}_{r} \\ \tilde{\mathbf{f}}_{f} \\ \tilde{\mathbf{f}}_{b} \end{cases} + \begin{cases} \mathbf{0} \\ \mathbf{0} \\ \mathbf{g}_{b} \end{cases}$$

4.1.3 Dual Craig-Bampton method

The Dual Craig-Bampton method (DCB) [23] is a newer method (2004) and uses the same approximation basis as the Rubin and MacNeal methods in Eq. (15). However, where the Rubin and MacNeal methods employ the second transformation, the DCB method keeps the interface forces as part of the generalized DOF. Hence, the assembly procedure is later different compared to the other three methods. The reduction basis is written as:

$$\left\{ \begin{array}{c} \mathbf{u} \\ \mathbf{g}_b \end{array} \right\} \approx \left[\begin{array}{c} \boldsymbol{\Phi}_r & \boldsymbol{\Phi}_f & \boldsymbol{\Psi}_r \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{\eta}_r \\ \boldsymbol{\eta}_f \\ \mathbf{g}_b \end{array} \right\} = \mathbf{R}_{DCB} \, \mathbf{q}_{DCB}$$
(24)

The substructure equations of motion are written as:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \left\{ \begin{array}{c} \ddot{\mathbf{u}} \\ \ddot{\mathbf{g}}_b \end{array} \right\} + \left[\begin{array}{c} \mathbf{K} & -\mathbf{A}^{\mathrm{T}} \\ -\mathbf{A} & \mathbf{0} \end{array} \right] \left\{ \begin{array}{c} \mathbf{u} \\ \mathbf{g}_b \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{f} \\ \mathbf{0} \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{0} \\ -\mathbf{u}_b \end{array} \right\}$$
(25)

The second row in Eq. (25) is added to enforce the compatibility during assembly. When Eq. (24) is inserted in Eq. (25) the following Dual Craig-Bampton reduced equations of motion are obtained:

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{r,bb} \end{bmatrix} \begin{pmatrix} \ddot{\boldsymbol{\eta}}_r \\ \ddot{\boldsymbol{\eta}}_f \\ \ddot{\mathbf{g}}_b \end{pmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{\Phi}_{r|b}^{\mathrm{T}} \\ \mathbf{0} & \Omega_f^2 & -\mathbf{\Phi}_{f|b}^{\mathrm{T}} \\ -\mathbf{\Phi}_{r|b} & -\mathbf{G}_{r,bb} \end{bmatrix} \begin{pmatrix} \boldsymbol{\eta}_r \\ \boldsymbol{\eta}_f \\ \mathbf{g}_b \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Phi}_r^{\mathrm{T}} \mathbf{f} \\ \boldsymbol{\Phi}_f^{\mathrm{T}} \mathbf{f} \\ \boldsymbol{\Psi}_r^{\mathrm{T}} \mathbf{f} \end{pmatrix} - \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}_b \end{pmatrix}$$
(26)

4.2 Assembly

The assembly of substructures consists of three main equations [14]:

$$\mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{f} + \mathbf{g}$$

$$\mathbf{B}_{b} \mathbf{u} = \mathbf{0}$$

$$\mathbf{L}_{b}^{T} \mathbf{g} = \mathbf{0}$$
(27)

where the first equation represents the assembled equation of motion and where \mathbf{M} , \mathbf{C} and \mathbf{K} are block diagonal matrices containing the (reduced) mass, damping and stiffness matrices of the substructures. The block vectors \mathbf{u} , \mathbf{f} and \mathbf{g} contain the (reduced) substructure displacements, external and connecting forces. They are defined as:

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(n)} \end{bmatrix}$$
(28)

The second and third equations represent the compatibility and equilibrium conditions. The block vectors \mathbf{B}_b and \mathbf{L}_b contain the substructure matrices $\mathbf{B}_b^{(s)}$ and $\mathbf{L}_b^{(s)}$, which connect the substructure interface DOF with a global set of interface DOF for the compatibility and equilibrium conditions. It also holds that \mathbf{L}_b is the nullspace of \mathbf{B}_b . This is a useful property, which leads to the derivation of a primal assembled system with the localized λ -method. They are defined as:

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \mathbf{L}_{b} \\ \mathbf{L}_{b}^{T} \mathbf{M}_{bi} & \mathbf{L}_{b}^{T} \mathbf{M}_{bb} \mathbf{L}_{b} \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{q}}_{i} \\ \ddot{\mathbf{q}}_{b} \end{pmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \mathbf{L}_{b} \\ \mathbf{L}_{b}^{T} \mathbf{K}_{bi} & \mathbf{L}_{b}^{T} \mathbf{K}_{bb} \mathbf{L}_{b} \end{bmatrix} \begin{pmatrix} \mathbf{q}_{i} \\ \mathbf{q}_{b} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{i} \\ \mathbf{L}_{b}^{T} \mathbf{f}_{b} \end{pmatrix}$$
(29)

where \mathbf{M}_{ii} , $\mathbf{M}_{ib} = \mathbf{M}_{bi}^{T}$, \mathbf{M}_{bb} , \mathbf{K}_{ii} , $\mathbf{K}_{ib} = \mathbf{K}_{bi}^{T}$ and \mathbf{K}_{bb} are the block diagonal matrices containing the substructure elements. These matrices contain the inner i and boundary b elements of the mass and stiffness matrices. More details about the assembly procedure can be found in [26] and [14]. This type of assembly is sometimes referred to as a primal stiffness assembly and is used in the Craig-Bampton, MacNeal and Rubin reduced models. The assembly procedure for the Dual Craig Bampton reduced models is slightly different, since the coupling is based on interface forces, and is as follows:

$$\begin{bmatrix} \mathbf{M}_{ii} & -\mathbf{M}_{ib} \mathbf{B}_b^T \\ -\mathbf{B}_b \mathbf{M}_{bi} & \mathbf{B}_b \mathbf{M}_{bb} \mathbf{B}_b^T \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{q}}_i \\ \ddot{\boldsymbol{\lambda}} \end{pmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & -\mathbf{K}_{ib} \mathbf{B}_b^T \\ -\mathbf{B}_b \mathbf{K}_{bi} & \mathbf{B}_b \mathbf{K}_{bb} \mathbf{B}_b^T \end{bmatrix} \begin{pmatrix} \mathbf{q}_i \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_i \\ -\mathbf{B}_b \mathbf{f}_b \end{pmatrix}$$
(30)

This type of assembly is also referred to as a primal flexibility assembly [26, 21].

4.3 Results

The comparison of the four methods is presented, where the first 20 mode shapes from both the legs and the rest of the structure are taken. The washing machine (without legs) is reduced from 228258 DOF to 284 DOF (20 from the interface modes and 264 from the boundary DOF) and the legs are reduced from 786 DOF to 157 DOF (20 from the interface modes and 137 from the boundary nodes). In total, the assembled model is reduced from 231402 DOF to 304 DOF with the Crag-Bampton, MacNeal and Rubin methods and to 649 DOF with the Dual Craig-Bampton method. The difference is due to the assembly procedure (Eq. (29) and Eq. (30)). A further reduction is possible with the use of interface reduction [12, 4]. The comparison of the results for the four methods is given in Table 3. The resulting relative errors compared to the FEM model are shown in Figures 6 and 7. It is clear that the Craig-Bampton method is accurate to within ~ 1 % compared to the FEM method for all the modes, except for mode 15. The other three methods have a similar accuracy due to the same reduction basis (Eq. (15) and Eq. (24)). Their accuracy is within 3.5 %, compared to the FEM method. Note, that the MacNeal method has slightly less accurate results due to neglected residual mass $\mathbf{M}_{r,bb}$ (Eq. (26)). The computation times needed for the initial computation of the whole structure as well as the later re-analysis of different legs and their assembly with the remaining structure are shown in Table 4.



Figure 6: Shore 70A legs relative errorsFigure 7: Shore 85A legs relative errors

From Table 4 it is clear that the initial preparation times using model reduction techniques are longer compared to the classic FEM approach. It can be observed that the dynamic modes are based on FEM modal analysis, which also represents the basis for model reduction. Note that the advantage of componentmode synthesis techniques is not to effectively perform the modal analysis of the whole structure, but to reduce the system DOFs. This is especially the case when explicit dynamic analyses are conducted or when recalculations of subsystems are performed. From Table 4 it is clear that the initial preparation times using model reduction techniques are longer compared to the classic FEM approach. It can be observed that the dynamic modes are based on FEM modal analysis, which also represents the basis for model reduction techniques. Note, that the advantage of component-mode synthesis techniques is not to effectively perform the modal analysis of the whole structure, but to reduce system DOFs. This is especially the case whenever explicit dynamic analyses are conducted or when recalculations of subsystems are performed. The later is shown in Table 4, where it is demonstrated that the re-analysis of the leg structure and the assembly with the remaining structure is ~ 90 times faster than using the

	Frequency [Hz]							
	Shore 70A				Shore 85A			
#	CB	MN	R	DCB	CB	MN	R	DCB
1	29.66	28.93	28.99	28.99	31.46	30.53	30.52	30.52
2	35.34	34.45	34.39	34.39	36.92	36.05	35.99	35.99
3	44.69	44.69	44.49	44.49	44.72	44.72	44.72	44.72
4	53.30	53.14	53.10	53.11	53.31	53.25	53.25	53.25
5	57.29	54.78	54.56	54.58	59.88	58.07	57.80	57.83
6	60.66	60.14	60.10	60.11	60.71	60.13	60.10	60.10
7	61.46	60.67	60.60	60.60	62.54	61.10	61.09	61.03
8	67.14	65.76	65.57	65.61	67.86	66.63	66.45	66.49
9	78.62	78.41	78.38	78.38	78.65	78.50	78.47	78.48
10	82.00	81.68	81.18	81.36	83.03	82.89	82.51	82.66
11	83.08	82.92	82.87	82.88	83.14	83.00	82.97	82.97
12	84.70	84.09	83.92	83.95	84.91	84.37	84.18	84.22
13	85.27	84.72	84.68	84.69	85.36	84.95	84.85	84.89
14	88.58	88.47	88.35	88.39	88.35	88.32	88.24	88.27
15	92.07	89.97	89.85	89.88	91.57	90.10	89.95	89.99
16	96.66	96.84	96.06	96.41	98.73	100.27	98.58	99.44

Table 3: Natural frequencies of the two leg configurations computed with four model reduction methods.

Table 4: Computation times of washing machine (WM) modal analysis, the reanalysis of legs and their assembly with the remaining structure and with the classical FEM approach.

Time [s]				
CB	MN	R	DCB	Full model (FEM)
98.201	98.545	98.545	98.545	
158.955	170.098	170.098	170.098	
2.331	2.467	3.012	2.373	
0.082	0.155	0.155	0.155	
0.626	1.119	1.119	1.174	
0.151	0.157	0.235	0.098	
0.209	0.209	0.209	0.922	
260.555 1.068	$272.750 \\ 1.640$	273.373 1.718	274.192 2.294	99.529
	CB 98.201 158.955 2.331 0.082 0.626 0.151 0.209 260.555 1.068	CB MN 98.201 98.545 158.955 170.098 2.331 2.467 0.082 0.155 0.626 1.119 0.151 0.157 0.209 0.209 260.555 272.750 1.068 1.640	$\begin{tabular}{ c c c c c } \hline Tim \\ \hline CB & MN & R \\ \hline 98.201 & 98.545 & 98.545 \\ 158.955 & 170.098 & 170.098 \\ 2.331 & 2.467 & 3.012 \\ \hline 0.082 & 0.155 & 0.155 \\ 0.626 & 1.119 & 1.119 \\ 0.151 & 0.157 & 0.235 \\ \hline 0.209 & 0.209 & 0.209 \\ \hline 260.555 & 272.750 & 273.373 \\ 1.068 & 1.640 & 1.718 \\ \hline \end{tabular}$	Time [s] CB MN R DCB 98.201 98.545 98.545 98.545 158.955 170.098 170.098 170.098 2.331 2.467 3.012 2.373 0.082 0.155 0.155 0.155 0.626 1.119 1.119 1.174 0.151 0.157 0.235 0.098 0.209 0.209 0.209 0.922 260.555 272.750 273.373 274.192 1.068 1.640 1.718 2.294

classic FEM approach. The fastest computation is with the Craig-Bampton method. The Rubin, MacNeal and Dual Craig-Bampton methods have slightly longer computation times, which is mainly due to the slightly longer computa-

tion times necessary to obtain the static modes. Moreover, the dynamic modes (the free-interface modes) require longer computation times compared to the Craig-Bampton method, where the fixed-interface modes include a smaller number of DOFs. The slowest model-reduction method is the Dual Craig-Bampton method, which is due to the slightly larger number of the reduced equations of motion that results in longer assembly times. Note, however, that the reduction itself is faster compared to Rubin and MacNeal methods due to simpler definition of reduced system matrices (Eq. (26)). It can also be observed that the MacNeal method computation time is slightly faster compared to Rubin and Dual Craig-Bampton methods, which is due to neglected mass residual term $\mathbf{M}_{r,bb}$ in Eq. (20). It should also be stated that whenever the boundary DOFs are changed during re-analysis the free-interface methods (MacNeal, Rubin and Dual Craig-Bampton) are faster, since the dynamic modes (the free interface modes) do not need to be recomputed. It can therefore be concluded that the substructuring techniques enable rapid recalculation times and therefore efficient optimisation analyses with accurate results.

5 Conclusions

An innovative numerical leg model is presented and validated experimentally. The presented approach introduces the so-called shear modulus correction factor k_{corr} that enables modelling of the contact conditions between the legs and the ground. The modelling is based on the measured dynamical properties of the whole washing machine structure, and therefore the measurement of the contact area is not needed [20]. It is demonstrated that the proposed identification process is reliable and accurate and may be applicable to leg structures with different hardnesses. The analysis of the two leg configurations shows that the leg stiffness increases for the first and second natural frequencies. Increasing the first natural frequency by 2 Hz (120 rpm) can significantly reduce the vibration and noise of the washing machine. Here it is shown that this can be achieved with leg modifications, in particular the leg rubber hardness, which influences the tangential stiffness of the system and therefore the system dynamics. In addition to the classic FEM analysis, substructuring techniques are proposed, where the legs and the cabinet are treated as substructures. This provided a better insight into the local dynamics and their effect on the global behaviour. In combination with the substructure assembly methods, four different modelreduction methods show an alternative to the FE numerical analysis. They reduce the number of degrees of freedom from ~ 230000 to ~ 300 and show good matching of the natural frequencies with the FE model. The Craig-Bampton method matches the first 16 frequencies with an error of less than 1.2 % error, except for one frequency with a 2.5 % error, whereas the other three methods have a slightly larger error with a maximum of 3.5 %. The assembly procedure enables a reanalysis of the modified substructure (i.e., the legs of the washing machine), while the remaining structure is already determined and therefore computed only once. This offers a better management of the computation time and the efficiency. In addition, the substructuring techniques can be exploited in further analyses, especially in explicit dynamics.

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